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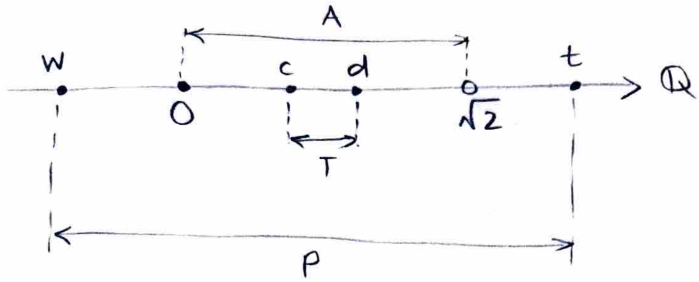
Exercise 1.5.14

a)



Let $A = \mathbb{Q} \cap (0, \sqrt{2})$

there are numbers in $! \mathbb{Q} \cap (0, \sqrt{2})$ w, t where $w < t$, such that $A \subseteq [w, t]$, and hence A is an interval

for numbers c, d in A, where $c < d$, we have $T = [c, d]$ such that $T \subseteq A$, and hence A is bounded.

b) A total ordered field $(F, \leqslant)$ is complete iff all non empty $A \subseteq F$ that have an upper bound also have a supremum (lowest upper bound).

All ordered fields contain a copy of $\mathbb{Q}$, and $\mathbb{Q}$ is dense $\rightarrow$ for any $x < y$, there is a q such that $x < q < y$.

If we have $(F, \leqslant)$, and $A = [a, b]$, but b is not ~~that~~ in F that an element of F. We will have some number t that is an upper bound of the interval $[a, b)$ in F. No matter how close t is to b, there will always be a number q such that $b < q < t$. Hence A has no supremum, but has an upper bound $\Rightarrow$ F cannot be complete.